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It may be remarked that d is a mean proportional between a and m , l is a mean proportional between $3a$ and m , and l is the diagonal of a cube whose face-diagonal is s and edge d .

(6). To determine a point equidistant from all the vertices.

LK bisects MH at O .

In the rectangle of which GH and MN are opposite sides, GN bisects MH , therefore passes through O and is itself bisected at O .

So with all the longest diagonals.

Hence O is equidistant from all the vertices.

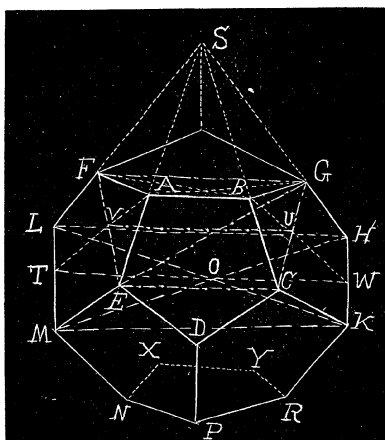
COROLLARY 1. O is equidistant from all the edges.

COROLLARY 2. O is equidistant from all the faces.

(7). To compute the volume.

This could be done by conceiving the dodecahedron composed of twelve equal pyramids with O as their common vertex and the faces for bases. But another method will be adopted.

The dodecahedron is composed of a cube one of whose faces is $EFGC$ and six equal truncated triangular prisms, one of which has AB , FG , and EC for its lateral edges. A right section of this truncated prism is a triangle with base equal to EF , and altitude, the perpendicular from A to UV . Since $ABXY$ is a rectangle equal to $LMKH$, this perpendicular is one-half the difference between $AX(=LH)$ and an edge of the cube, and, therefore, equals $\frac{1}{2}a$.



Hence the area of the right section is $\frac{a^2}{8}[\sqrt{5}+1]$, and the volume of the truncated prism is $\frac{a^2}{8}[\sqrt{5}+1]\frac{a+a[\sqrt{5}+1]}{3}$.

The sum of the volumes of six such solids is $\frac{a^3}{4}[7+3\sqrt{5}]$.

Volume of cube $= \frac{a^3}{4}[8+4\sqrt{5}]$.

Volume of dodecahedron $= \frac{a^3}{4}[15+7\sqrt{5}]$.

134. Proposed by J. C. GREGG, A. M., Superintendent of Schools, Brazil, Ind.

If $ABCD$ is a quadrilateral circumscribing a circle, show that the line joining the middle points of the diagonals AB , CD passes through the center of the circle.

Solution by HARRY S. VANDIVER, Bala, Penna.

We will use quadrilinear coordinates denoting the equation of the four

sides $\alpha=0$, $\beta=0$, $\gamma=0$, $\delta=0$, and the *lengths* of the corresponding sides by a , b , c , and d . Let radius of circle be r . Then the equation of the line joining the middle points of the diagonals is $a\alpha - b\beta + c\gamma - d\delta = 0 \dots (1)$ (Cf. Salmon's *Conics*, page 54, Ex. 5).

Putting $\alpha=\beta=\gamma=\delta=r$ we obtain $r(a-b+c-d)=0$, which is satisfied since $a+c=b+d$.

137. Proposed by J. W. YOUNG, Oliver Graduate Scholar in Mathematics, Cornell University, Ithaca, N. Y.

A right cone has its vertex in a horizontal plane, its axis being perpendicular to the plane. A string has one extremity attached to a point on the cone. The other extremity, P , of the string is kept in the plane, and the string is then wound around the cone, without being allowed to slip. Show that the spiral generated by P cuts all straight lines through the vertex at the same angle.

Solution by the PROPOSER.

Let P , P' be two points on the spiral; Q , Q' the corresponding points in the path of the string around the cone; N , N' the points where the perpendiculars from Q , Q' to the plane through the vertex O of the cone, cut the plane.

The right-angled triangles QNO , $Q'N'O$ have the angles QON and $Q'ON'$ equal; hence they are similar.

$$\therefore \frac{QN}{ON} = \frac{Q'N'}{ON'} \dots (1).$$

Again, since the string must not slip, it makes a constant angle with the plane.

$\therefore \triangle QNP$ is similar to $\triangle Q'N'P'$.

$$\therefore \frac{PN}{QN} = \frac{P'N'}{Q'N'} \dots (2).$$

From (1) and (2),

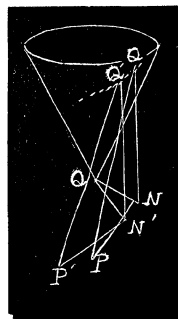
$$\therefore \frac{PN}{ON} = \frac{P'N'}{ON'} \dots (3).$$

But the triangles ONP and $ON'P'$ are right-angled at N and N' (PN , $P'N'$ being the projections to tangents to the circular cone). From (3),

$\therefore \triangle ONP$ is similar to $\triangle ON'P'$.

$\therefore \angle OPN = \angle OP'N'$.

Observing that PN and $P'N'$ are normals to the spiral, the last equation states that the normals make a constant angle with rays through O . Q. E. D.



AVERAGE AND PROBABILITY.

90. Proposed by WALTER H. DRANE, Graduate Student, Harvard University, Cambridge, Mass.

During a heavy rain storm a circular pond is formed in a circular field. If a man undertakes to cross the field in the dark, what is the chance that he will walk into the pond? [From Byerly's *Integral Calculus*.]

I. Solution by B. F. FINKEL, A. M., M. Sc., Professor of Mathematics and Physics, Drury College, Springfield, Mo.

Let O be the center of the circular field, and R its radius; C the center of